Examiners' Report

Principal Examiner Feedback

January 2022

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 02

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Introduction

There were some very good responses in this paper with some candidates demonstrating a very high level of mathematical ability.

Questions which discriminated particularly well between differing abilities were:

- Question 2 [trigonometrical equations]
- Question 4(c) [3D trigonometry]
- Question 6(b) [Arithmetic series]
- Question 7 (a) and (b) [Roots of quadratic equations]
- Question 8 (b) and (d) [Solution of equations by graphical methods]
- Question 9(d) [Vectors]

Question 1

Both parts of this question were a source of 4 straightforward marks for many candidates who were able to make a confident start to the paper.

(a) It was clear that a small number of candidates did not realise the question was in radians and as such quoted and used the formula $\frac{\theta}{360}\times\pi r^2$ with the angle of 1.2 radians which scored zero marks.

For those who had identified that the question was in radians, very occasionally candidates had an spurious π in their equation which clearly came from mis-remembering the formula or not simplifying $\frac{\theta}{2\pi}\times\pi r^2$ correctly.

Most candidates approached this question directly, either quoting or miss-quoting the formula, however some of those candidates who tried to work this out as a proportion of the circle generally did not gain both marks due to errors in working.

b) This part was answered less well than part (a) although those candidates who approached this directly with the correct formula for the perimeter $P = 2r + r\theta$ were successful in gaining both marks. It we very rare to see a correct method like this and then an incorrect final answer. Students should be mindful of checking their calculations, especially when using a calculator.

As expected, when a candidate did not score full marks, a significant number found the correct arc length, but clearly had not read the question and left this as their final answer, again a timely reminder here for students to read the question carefully and answer the question given not the question they think has been asked.

Again, as in part (a) the spurious π was sometimes used to calculate the arc length, which lost the method mark, regardless whether the candidate found the perimeter as their final answer or the arc length.

Question 2

On the whole this was not a well answered question for the majority of candidates who did not spot the obvious simple method which was to use the identity $\tan A = \dfrac{\sin A}{\cos A}$

As the formulae for cos (A + B) and sin (A + B) are given on the formulae sheet,[as indeed is the identity for tan A] many candidates tried to use these identities. Most who attempted this (and there were many) made errors in manipulation and were unable to pick up any marks. It is of course possible to solve the equation in this way, but there is a great deal of careful work for the award of just 5 marks, which in itself should have been a hint that the solution is simpler than many candidates thought at first. The few who did carry out the easier way of solving the equation, by write the equation in terms of $\tan(2\theta - 20)^{\circ}$ did on most occasions score all the marks.

The best answers seen were those where candidates replaced $(2\theta - 20)$ with 'x 'to simplify the equation, and then rearrange to write in terms of tan'x 'and then bring the $(2\theta - 20)$ back in to solve the equation for the specified region. .

There were a few candidates who only found one solution (40°) without showing any indication there would be more than one solution in the range given.

It is pleasing to note that there was no evidence of candidates attempting to work in radians.

Question 3

Overall, his question was very well answered with every candidate realising that integration was required.

Most were able to find the two x-values where the curve intersects with the x-axis. However, there was a small minority who only gave the value of +3, which lost them the first B mark and also the final accuracy mark. There were also a few candidates who did not find any x-values at all, and simply carried out an indefinite integral. These candidates clearly knew that area under a curve is found by integration but did not know how to find the limits for their integration.

Just a few candidates misunderstood the question and attempted to find the volume of revolution, but these attempts were rare.

The process of integration was very well done in the question; almost all candidates could integrate the expression with ease, gaining the M1 and A1.

Substitution of their limits into the integral was usually shown, thus allowing the method mark even when the limits were incorrect. Centres should note that they must encourage candidates to show this line of substitution in every solution because we reward sight of correct substitution even into an incorrect integrated expression but will not award the method mark if this substitution is not seen. There were a few candidates who did lose out on the method due to this.

Question 4

(a) Most candidates found AC first, then found FC by 2D Pythagoras theorem, very few candidates applied *3D* Pythagoras theorem to find the diagonal length FC of a cube in one step. This part was easily accessible for virtually every candidate who scored full marks. Just a few lost the A mark because they provided the decimal value of the length instead of the exact length which the question asked for.

(b) Once again, the majority of candidates also answered well in this part. Most used one of sine, cosine or tangent ratios in triangle *AFC* to successfully find the correct angle *FCA*. A few candidates were seen applying cosine rule in a right-angle triangle by following through their lengths of *AC* and *FC* to find angle *FCA*.

(c) This part proved to be the most challenging part of the question, a lot candidates managed to scored full marks in part (a) and (b), but scored no marks at all in this part. Many candidates were confused about which angle between the plane BFH and plane BHC was. It is clear that there is a weak understanding of how to calculate the angle between two planes. Completely correct solutions were not often seen.

A few able candidates spotted that a very simple way to find the required angle was to use triangle *FGX* where *X* is the midpoint of *BH* because the required angle is supplementary to angle *FCX*.

Centres should note that successful candidates took the trouble to annotate and draw on the given sketch as well as draw labelled thumbnail sketches of the triangles under consideration.

Question 5

A good number of candidates found this question very accessible, and it was a source of a good many marks for most.

(a) The majority understood well that in order to find the values of t when P is instantaneously at rest means setting the expression for $v = 0$ and then solving the 3 term quadratic to find the two values of t. Most candidates were able to easily find the two correct values for t. A few candidates made trivial and carless errors whilst factorizing the 3TQ, but usually reached one correct value for t. Correct answer found from incorrect method however, scored no marks.

(b) Most recognized the need to differentiate to get acceleration in part (b) and did so correctly. However, writing down a correct inequality for t after this proved a step too far for some. A significant number of candidates only scored M1 for $\frac{dv}{dt} = 6t - 23,$ then A0 for either trying to solve $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ or giving t in the incorrect inequality sign. It was surprising to see that some candidates were not sure when to switch sign while solving a linear inequality.

A correct differentiated expression for $\frac{\mathrm{d}v}{\mathrm{d}t}$ was required for first M mark.

(c) This was the least well answered part of the question for some candidates who often did not realise that to find the distance they needed (as a first step) to integrate v. Candidates who did realise this often integrated accurately but did not always include + c scoring M1A0M0A0. Some candidates with + c did not always appreciate the need to put $t = 8$ and $s = 26$ into their integrated function in order to find the constant of integration.
Only a small minority achieved 4 out of 4 marks here..

<u>Question 6</u>

(a) Almost every candidate found the correct sum of 20 terms using the given expression for S_n.

(a) All of the difficulty in this question occurred in part (b) where a fully correct solution was a rare sight indeed and many candidates did not attempt this part of the question at all.. Getting started was the main issue here. Very few followed Method 1 in the mark scheme. Almost all students who attempted part (b) chose to follow the alternative approach by setting up simultaneous equations for A and B and solving these, although there were many errors seen with a great deal of confusion between the nth term and the sum.

The few successful candidates who applied Method 2 in the scheme often listed $S_1 = A + B$, $S_2 = 3A + 2B$ but realised that the first term of the series is $A + B$, the second term is $2A + B$ and solved these two simultaneous equations easily finding the correct value of $A = 4$ and $B = 1$.

Some candidates used the sum of 20 terms found in (a), but very rarely did they achieve the correct expression in terms of A and B which is $210A + 20B = 860$

(c) Surprisingly, part (c) was often attempted successfully without having managed to answer part (b). There did appear to be some misunderstanding as to the difference between T_n and S_n in this part especially when trying to set up the linear equation to solve for n. Some candidates were confused by the letters thinking that T_n is the nth term of the second series, setting up $T_n = 7 + (n-1) \times 4$, and then setting up $S_n = \frac{n}{2}(2 \times 7 + (n-1)4)$ as the sum of the second series. These candidates could score no marks in part (c).

Some candidates found T_n correctly and equated their $T_n = S_n + 252$. They did not realise however that S_n in part (c) is the same S_n in part (a) and that they could have substituted $n(3+2n)$ to their S_n in part (c). Instead, they tried to find S_n again by using first term 5 and common difference 4 which they found in part (b). This was a longer approach, but those candidates who persevered with this method eventually reached the same sum $3n + 2n^2$ and solved the equation successfully.

Question 7

Overall, this question was not answered well. Many candidates made simple errors in manipulating the algebra, which affected both part (a) and part (b).

(a) The vast majority of candidates were able to find the sum and product of roots of the equation f $(x) = 0$ with little issue. However, the sum of the roots of g $(x) = 0$ caused many problems as it was common to see $\alpha + \beta = -\frac{37}{14}$ incorrectly stated instead of the required $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{37}{14}$. Many candidates were successful at manipulating the algebra to obtain $\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ for the first M mark, but many did not equate this to the value of $-\frac{37}{14}$.

Substitution of $-\frac{p}{2}$ and $\frac{q}{2}$ was usually successfully obtained by the vast majority of candidates although a small number processed $\left(-\frac{p}{2}\right)^2$ as $-\frac{p^2}{2}$ and as no initial correct substitution was seen the mark was lost.

A common error for candidates at this stage, who did not find correct algebra, was to simply substitute $\alpha^2 = -\frac{p}{2}$ and $\beta^2 = \frac{q}{2}$ into $\frac{\alpha^2 + \beta^2}{\alpha\beta}$ as well as setting their equation equal to zero instead of their $-\frac{37}{14}$.

Most candidates who correctly manipulated the algebra and correctly substituted arrived at the correct 3TQ and were able to finish this part of the question with ease.

For those candidates who did not correctly manipulate the sum of roots ready for substitution, it was not uncommon for candidates to arrive at linear equation and thus lose the subsequent marks for solving a quadratic, which they could not obtain.
In terms of solving the quadratic, a large portion showed correct working. The most common approach was factorisation into 2 factors, but we also observed correct use of quadratic formula and completing the square to solve, which was pleasing to see.
Centres are reminded that those candidates who rely on their calculators to solve such quadratics without showing a method, will lose the method mark if their 3TQ is incorrect. We observed this several times in this question.

(b) This part was less well answered than part a) with a number of candidates only scoring one mark, either for $\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta)$ or $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$. A frequently seen error was $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

The most able of candidates were able to embed the second of the 2 expressions into the first with ease so $\sqrt{(\alpha + \beta)^2 - 4\alpha\beta} \times (\alpha + \beta)$ was seen; those candidates who used this approach were generally successful in substitution of their *p* and *q*.

Although not common, but very successful, some candidates calculated the value for $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ before substitution into $(\alpha - \beta)(\alpha + \beta)$. The vast majority of those candidates who made significant progress on this question were able to use their answer for *p* and *q* to evaluate $(\alpha - \beta)(\alpha + \beta)$ albeit arriving at an incorrect final answer.

Although this was a 'show' question we did not have the usual issue of candidates "fudging" answers – many realised they did not have the required information from part (a) so simply did not make much progress on this part.

Question 8

(a) This was the best answered part of question 8. The vast majority of candidates were able to find the correct missing values in the table to one decimal place. However there were a small minority who did not find all of these values correctly or round them correctly. We insist on correct rounding in this topic.

(b) Unusually in this topic, it was disappointing to see so many incorrectly plotted points in this part of the question to lose the first B mark. Many candidates plotted the third value in the table (0.8, 0.3) at the incorrect position [usually $(0.8, -0.3)$]. As this was a given point, there should really have been no error in plotting this point. Sometimes the 0.8 was incorrectly positioned, usually by placing it two squares before 1.0 instead of 4 squares before it.

The second B mark was awarded for a smooth curve, through **all** of the points. When this mark was lost it was usually because candidates did not draw a curve at the maximum point and left it as a sharp point. Some lost this mark by simply joining points together with straight lines.

(c) This was a well answered part. Candidates choose the correct formula of cos (A + B) from the formulae page and using the identity of $\sin^2 A + \cos^2 A = 1$ found the required identity of $\cos 2A = 1 - 2\sin^2 A$ correctly.

(d) The majority of candidates did not score any marks in this part of the question. Those who did, were most successful when they substituted from $2\sin^2 x = 1 - \cos 2x$ into f (x). There were many candidates who attempted to substitute into the y equation instead and try to match up the f (x). This was not done very successfully on most occasions, mostly because candidates did not realise they needed to use part (c), usually resulting in no marks at all in this part, or maybe just the first method mark for a correct substitution. There were a just a few correct solutions seen, where candidates found the correct linear graph and plotted it correctly to find the two correct roots for f (x) = 0.

Question 9

Parts (a) and (b) were well answered. The two marks in each were scored by most every candidate, with only a few slips with signs when there were errors. Centre should note carefully however, that the M mark is awarded for a correct vector statement so the marking method for part (a) is as follows;

M1 - $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$

A1 - $\overrightarrow{AB} = -2\mathbf{a} + 4\mathbf{b}$

It is therefore very important that candidates write out their vector statements as a starting point.

(c) This part was completed well by around half of the candidates. Usually they deduced a correct vector statement for $\overrightarrow{OQ}$ using the ratio $OQ : QP = 3 : 1$ and were able to find a correct vector statement for $\overrightarrow{AQ}$. As the algebra was straight forward, when candidates realised what to do, it was usually completed with no errors.

(d) This was very poorly answered by almost all candidates. Very few scored full marks here. Some could see they needed $\overrightarrow{OR}$ written in terms of another parameter and then needed a second vector with a different parameter in order to equate coefficients. There were many who found $\overrightarrow{OR}$ in terms of another parameter, but then did not do anything else worthy of credit. Those who set up correct and valid equations, with 2 constants, generally knew they had to equate coefficients of **a** and **b** and were able to solve the resulting equations successfully. Some who achieved the correct values of the constants could not always interpret their answer as a correct ratio of lengths. However, as previously stated, most had no idea what statement to write down in order to compare coefficients. There were also too many candidates who did not attempt this question at all.

Question 10

There were many candidates who only answered part (a) to (c) and it is unclear whether this was due to running out of time or not having the knowledge required for (d) and (e).

(a) Most candidates were successful in identifying the asymptotes when answering parts (i) and (ii), although a number of students, who scored the second mark in (a)(ii) had $y = \frac{1}{2}$ for

When (i) and (ii) were not clearly labelled, some candidates stated the equations the wrong order so implying that $x = -4$ was parallel to the x-axis and that $y = 2$ was parallel to the

y-axis. It is worth reminding candidates to label their answers clearly with question parts, to ensure that they are credited for their work.

Although uncommon, a very small number of candidates gave their answers as a number rather than an equation, again resulting in zero marks in this part as this is not an equation.

(b) Many candidates were able to find the correct coordinates in the vast majority of cases. Very occasionally, values were not expressed as a coordinate pair, and although the question asked for coordinates, we also gave full credit to answers given in the form

$y = -\frac{1}{4}$ and $x = \frac{1}{2}$

(c) The successful completion of the question was dependant on candidates' answers to parts (a) and (b) for which full follow through credit was given if incorrect asymptotes and/or intersections with the coordinates axes were used correctly.

Many candidates knew that the correct shape of the graph had two branches. However a small number did not draw the graph in the correct quadrants. When the graph was in the correct quadrants, with 2 branches and did not score the mark, it was generally because their curves turned back on themselves at the end of their branches. Centres should remind candidates of the importance of accurate branch drawing including the asymptotic nature of a reciprocal graph.

The vast majority of candidates were able to draw their solutions to part (a) thus gaining the follow through mark; however a limited number of candidates had the branches of their graph crossing their asymptotes, clearly demonstrating that they do not understand the nature of an asymptote.

Most candidates scored the final mark in part (c) as they had a curve intersecting their two coordinates found in part b). If not scored, candidates either did not draw both intersections on their graphs or, if they did, had their branch intersect only one.
For both of the final marks in this part, a number of accurate graphs were not labelled as requested, thus losing credit. Occasionally candidates sketched out axes with an integer scale shown; clearly, this does not specify where a curve crosses the axis if the value is rational, again losing the accuracy marks in this part.

(d) It was not uncommon to see accurate answers to parts (a) to (c), but no serious attempt made at the remaining parts. For those who did try to make progress, use of the quotient rule was generally accurate. A small number of candidates occasionally used $(2x-1)$ as their denominator. As expected, the general issue for those attempting this part were writing the terms in the correct order gaining the M mark but losing the A mark. It appears as those many candidates either ran out of time or did not know how to progress further than differentiating the given function. Many, having found $\frac{dy}{dx}$ correctly did not make the connection to setting it equal to the gradient of the tangent (m = 1) which needed to be extracted from the given equation of the line. It was not uncommon for weaker students to be seen substituting values into an expression rather than setting the expression equal to a given value. Those who did follow the correct process either expanded to form a 3TQ, or more elegantly used the squared nature of the equation to elicit values for x. If they solved the 3TQ then generally they were able to find the coordinates of P and Q correctly however occasionally, those who found x did not find values for y.

(e) This part was rarely attempted, possibly due to time issues or lack of an answer from part (d). By this stage those students who tried this part had little difficulty in realising they needed to substitute the coordinates found in (d) into the equation of the tangent to find k. Having found the equation, the values of find k_1 and k_2 were left embedded by some rather than being specifically identified. Previous errors unfortunately left a number of students unable to find k_1 or k_2 accurately, but they were able to gain the method mark in this part

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